LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2015**

MT 6608 - DISCRETE MATHEMATICS

Date : 20/04/2015 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

SECTION - A

ANSWER ALL QUESTIONS: **(10 x 2 = 20)**

1) Construct the truth table for $P \vee Q$.
2) Write the duals of (i) $(P \vee Q) \wedge R$ (ii) $(P \wedge Q) \vee T$.
3) Write down the min terms of P and Q.
4) Show that the equivalence $P \vee (P \wedge Q) \Leftrightarrow P$.
5) Define monoid and give an example.
6) Give an example of (i) finite cyclic monoid and (ii) infinite cyclic monoid.
7) Define Lattice.
8) Let $S = \{a, b, c\}$. Draw the diagram of $\langle \rho(S), \subseteq \rangle$.
9) Define Boolean Algebra.
10) Define Boolean homomorphism.

SECTION - B

ANSWER ANY FIVE QUESTIONS: **(5 x 8 = 40)**

11) Construct the truth table for $(P \to Q) \wedge (Q \to P)$.
12) Show that $((P \vee Q) \neg (\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \vee \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.
13) Obtain the principle disjunctive normal forms of (i) $\neg P \vee Q$ (ii) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.
14) Write the following sentences in the symbolic form:
    (i) Jack and Jill went up hill.
    (ii) If there is a flood then the crop will be destroyed.
    (iii) If either Jerry takes Calculus or Ken takes Sociology, then Lorry will take English.
15) Prove that the composition of semigroup homomorphisms is also a semigroup homomorphism.
16) Let $\langle L, \leq \rangle$ be a Lattice. Then prove that for any $a, b, c \in L$, the inequality $a \oplus (b * c) \leq (a \oplus b) * c$ holds.
17) Define (i) Lattice homomorphism and give an example:
    (ii) Lattice endomorphism
    (iii) Lattice automorphism
18) Let B be a Boolean algebra. Then prove that (i) $(a \oplus b)' = a' * b'$ (ii) $(a * b)' = a' \oplus b'$.

SECTION - C

ANSWER ANY TWO QUESTIONS: **(2 x 20 = 40)**

19) (a) Construct the truth table for the following statements (i) $\neg(P \vee Q) \Leftrightarrow (\neg P \vee \neg Q)$.

(ii) $P \wedge \neg P$.

(b) Obtain the p.d.n.f. of $(\neg P \rightarrow R) \wedge (Q \Leftrightarrow P)$. **(10+10)**

20) (a) Prove that for any commutative monoid $(M, *)$, the set of all idempotent elements of M forms a submonoid.

(b) Define sub semigroup and sub monoid and also give an example to each. **(12+8)**

21) (a) State and prove the four properties of Lattices.

(b) Define sub Boolean algebra. **(16+4)**

22) (a) Prove the following Boolean identities:

(i) $a \oplus (a' * b) = a \oplus b$

(ii) $a * (a' \oplus b) = a * b$

(iii) $(a * b) \oplus (a * b') = a$.

(b) Define the following: (i) semigroup (ii) semigroup isomorphism (iii) sub lattice (iv) direct product of two Boolean algebras. **(10+10)**

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